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when both (1) and (2) are satisfied, one has

$$\phi(x) = \tan ax, \qquad \psi(x) = \sec^2 ax,$$

which is the most general continuous solution, [also $\psi(x) \equiv 0$ and $\phi(x) = \text{const.}$—EDS.].

Also solved by ELIJAH SWIFT; partial solutions by E. B. ESCOTT, and H.L. OLSON.

2763 [1919, 170] Proposed by C. N. SCHMALL, New York City.

Show that the equation $ky - 2k^{1/3}a^{2/3}x + x^2 = 0$, where k is a variable parameter, represents a family of parabolas passing through a fixed point, and all having the same areas comprised between the curve and the x-axis.

Show, also, that the envelope of the family is the rectangular hyperbola whose equation is $xy = 2^5a^2/3^3$.

SOLUTION BY P. J. DA CUNHA, University of Lisbon.

Il est évident que l'équation donnée représente une famille de paraboles, et que toutes ces courbes jouissent de la propriété de passer par l'origine. L'abscisse du second point d'intersection de chacune des paraboles et de l'axe des x est $2k^{1/3}a^{2/3}$, de sorte que l'aire comprise entre la courbe et le même axe est

$$A = \frac{1}{k}\int_0^{2k^{1/3}a^{2/3}} (2k^{1/3}a^{2/3}x - x^2)dx = \frac{1}{k}\left[k^{1/3}a^{2/3}x^2 - \frac{x^3}{3}\right]_0^{2k^{1/3}a^{2/3}} = \frac{4}{3}a^2.$$

Finalement, l'équation de l'enveloppe s'obtenant par la règle connue, c'est à dire, en éliminant k entre l'équation donnée et celle-ci: $y - 2k^{-2/3}a^{2/3}x/3 = 0$, nous tombons sur l'équation

$$xy = \frac{2^5a^2}{3^3},$$

c. q. f. d.

After giving this same solution H. H. Downing, University of Kentucky, notes that the locus of the vertices of the parabolas is the equilateral hyperbola, $xy = a^2$.

The problem was also solved by C. A. BARNHART, A. M. HARDING, WILLIAM HERBERG, WILLIAM HOOVER, ARTHUR PELLETIER, and J. B. REYNOLDS.

2766 [1919, 171]. Proposed by N. P. PANDYA, Amreli, India.

Is it possible to find a harmonic series whose terms are positive integers such that the product of the first, second, seventh, and eighth terms is equal to the product of the third, fourth, fifth, and sixth terms?

SOLUTION BY A. M. HARDING, University of Arkansas.

Any harmonic series of eight terms may be written in the form

$$a, \quad \frac{a}{1+ak}, \quad \frac{a}{1+2ak}, \quad \cdots, \quad \frac{a}{1+7ak},$$

where a and k are constants, $a \neq 0$. If it is possible to find an harmonic series satisfying the conditions of the problem, then

$$a \cdot \frac{a}{1+ak} \cdot \frac{a}{1+6ak} \cdot \frac{a}{1+7ak} = \frac{a}{1+2ak} \cdot \frac{a}{1+3ak} \cdot \frac{a}{1+4ak} \cdot \frac{a}{1+5ak}.$$

That is, since $a \neq 0$,

$$(1+ak)(1+6ak)(1+7ak) = (1+2ak)(1+3ak)(1+4ak)(1+5ak),$$

or

$$16a^2k^2 + 112a^3k^3 + 120a^4k^4 = 0.$$

Supposing $k \neq 0$ and dividing by $8a^2k^2$, we have

$$2 + 14ak + 15a^2k^2 = 0;$$

whence

$$ak = \frac{-7 \pm \sqrt{19}}{15}.$$

Then

$$1 + ak = \frac{8 \pm \sqrt{19}}{15}, \quad \text{and} \quad \frac{a}{1 + ak} = \frac{15a}{8 \pm \sqrt{19}}.$$

That is, if a is a positive integer, the second term of the series is irrational. Hence, the only series which satisfies the condition of the problem is one for which $k = 0$; that is, a series in which the terms are all equal.

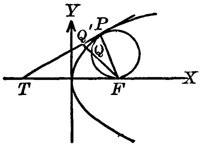
Also solved by H. L. Olson and Arthur Pelletier.

2779 [1919, 268]. Proposed by J. L. RILEY, Junior Agricultural and Mechanics College, Stephensville, Texas.

A parabola is placed with its axis horizontal; find the straight line of shortest descent from the curve to the focus.

I. Solution by A. M. Harding, University of Arkansas.

Construct a circle tangent to the axis of the parabola at F and tangent to the parabola at P[1]. The time of descent down all chords of this circle which pass through F is the same, so that the time down PF is the same as the time down any other chord QF, and is, therefore, less than the time down the focal radius $Q'F$.



II. Solution by H. S. Uhler, Yale University.

Let $p \equiv$ distance from vertex to focus, $g \equiv$ acceleration due to gravity, $s \equiv$ distance from any point (x, y) on the parabola and in the first quadrant to the focus $(p, 0)$, $t \equiv$ time of descent, and $\gamma \equiv$ angle which the straight line makes with the positive direction of the axis of x. From kinematics, $s = \frac{1}{2}at^2$ and $a = g \sin \gamma$; hence,

$$t^2 = \frac{2s}{g \sin \gamma}. \tag{1}$$

The polar equation of the parabola, with the pole at the focus, is

$$s = \frac{2p}{1 - \cos \gamma} = \frac{2p(1 + \cos \gamma)}{\sin^2 \gamma}. \tag{2}$$

Hence, from (1) and (2),

$$\frac{g}{4p} t^2 = \frac{1 + \cos \gamma}{\sin^3 \gamma}.$$

Differentiating,

$$\frac{gt}{8p} \cdot \frac{dt}{d\gamma} = \frac{-\sin^2 \gamma - 3(1 + \cos \gamma) \cos \gamma}{\sin^4 \gamma} = 0;$$

therefore,

$$2 \cos^2 \gamma + 3 \cos \gamma + 1 = 0$$

so that

$$\cos \gamma = -1 \quad \text{or} \quad \cos \gamma = -\tfrac{1}{2}.$$

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[1] If this construction is possible and if the tangent at P meets the axis in T, $TP = TF = x + p$, the equation of the parabola being $y^2 = 4px$. Hence to get the coördinates of P we have $(x + p)^2 = 4x^2 + 4px$ or $x = \frac{1}{3}p$. It follows that the triangle TPF is an equilateral triangle, each side equal to $\frac{4}{3}p$. The coördinates of the center of the circle will be p, $4p/3\sqrt{3}$, and its equation $(x - p)^2 + y^2 - 8py/3\sqrt{3} = 0$. If we eliminate x or y from this equation and the equation of the parabola we shallfind for the resulting equation two equal roots corresponding to p, and two complex roots, showing that the circle must lie entirely within the parabola—which is an important consideration in connection with Professor Harding's discussion of the problem.—Editors.